Grade 6 Math Unit 2-The Number System

UNIT OVERVIEW

In Grade 6, instructional time should focus on four critical areas. This unit addresses **Critical Focus Area # 2, Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers**.
(See Connections for explanation)

The work in this unit will address the following clusters:
- Understand ratio concepts and use ratio reasoning to solve problems.
- apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Students will be able to understand the value of positive numbers, negative numbers, and zero when used in real-world applications. Understand that integers are whole numbers and their opposites. Place rational numbers on a number line and ordered pairs onto a coordinate plane. Find the distance between two numbers on a number line or two ordered pairs which share the same x-coordinate or y-coordinate. Interpret inequality statements.

STANDARDS

CC_Common Core State Standards - Mathematics (2010) - Grade 6
Domain 6.RP Ratios and Proportional Relationships
Cluster Statement: ***Understand ratio concepts and use ratio reasoning to solve problems.***
Standard 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Domain 6.NS The Number System
Cluster Statement: ***Apply and extend previous understandings of multiplication and division to divide fractions by fractions.***
Standard 6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Cluster Statement: ***Compute fluently with multi-digit numbers and find common factors and multiples.***
Standard 6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.
Standard 6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Cluster Statement: ***Apply and extend previous understandings of numbers to the system of rational numbers.***

Standard 6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Standard 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

6.NS.6.a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

6.NS.6.b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

6.NS.6.c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Standard 6.NS.7 Understand ordering and absolute value of rational numbers.

6.NS.7.a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

6.NS.7.b Write, interpret, and explain statements of order for rational numbers in real-world contexts.

6.NS.7.c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

6.NS.7.d Distinguish comparisons of absolute value from statements about order.

Standard 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

CONTENT ELABORATIONS

6.NS.1	In grade 5 students divided whole numbers by unit fractions. Students connect this understanding by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students understand that a division problem such as 3 ÷ 2/5 is asking, "how many 2/5 are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder is 1/2. Therefore, 3 ÷ 2/5 = 7½, meaning there are 7 ½ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves. Students should also write contextual problems for fraction division problems. For example, the problem, 2/3 ÷ 1/6 can be illustrate with the following word problem: Susan has 2/3 of an hour left to make cards. It takes her about 1/6 of an hour to make each card. About how many can she make? The problem can be modeled using a number line. Start with a number line divided into thirds. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths. Circle each sixth. There are four sixths in two-thirds; therefore, Susan can make 4 cards. **MP.1, MP.2, MP.3, MP.4, MP.7, MP.8** should be emphasized.
6.NS.2	Procedural fluency is defined by the Common Core as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". In elementary grades students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm. This understanding is foundational for work with fractions and decimals in 7th grade. Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level. As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, "there are 200 thirty-twos in 8456" and could write 6400 beneath the 8456 rather than writing 64. **MP.2, MP.7, MP.8** should be emphasized.

6.NS.3	In fourth and fifth grades, students added and subtracted decimals. Multiplication and division of decimals was introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on the place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations. The use of estimation strategies supports students' understanding of operating on decimals. Example: - First students estimate the sum and then find the exact sum of 14.4 and 8.75. An estimate of the sum might be 14 + 9 or 23. Students may also state if their estimate is low or high. They would expect their answer to be greater than 23. They can use their estimates to self-correct. **MP.2, MP.7, MP.8** should be emphasized.
6.RP.3d	A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the quantity described in the numerator and denominator is the same. For example 12in./1ft. is a conversion factor since the numerator and denominator name the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as 1ft./12in. allowing for the conversion ratios to be expressed in a format so that units will "cancel". Example: How many centimeters are in 7 feet, given that 1 inch = 2.54 cm.? 7ft. x 12in./1ft. x 2.54cm./1in. = 7ft. x 12 x 2.54cm. = 213.36cm. **MP.1, MP.2, MP.4, MP.5, MP.7** should be emphasized.
6.NS.5	Students use rational numbers (fractions, decimals and integers) to represent real-world contexts and understanding the meaning of 0 in each situation. Example: - Use an integer to represent 25 feet below sea level. Use an integer to represent 25 feet above sea level. What would 0 (zero) represent in the scenario above? Solution: -25, +25, 0 would represent sea level **MP.1, MP.2, MP.4** should be emphasized.
6.NS.6a	In elementary grades, students worked with positive fractions, decimals and whole numbers on the number line. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical. Students recognize that a number and its opposite are equidistant from zero (reflections about the zero). The opposite sign (-) shifts the number to the opposite side of 0. For example, -4 could be read as "the opposite of 4" which would be negative 4. The following example, -(-6.4) would read as "the opposite of the opposite of 6.4" which would be 6.4. Zero is its own opposite.
6.NS.6b	Students worked with Quadrant I in elementary grades. As the x-axis and y-axis are extending to include negatives, students begin with the Cartesian Coordinate system. Students recognize the point where the x-axis and the y-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (-,+). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change in the x-coordinates from (-2, 4) to (2, 4) represents a refection across the y-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes.
6.NS.6c	Students are able to plot all rational numbers on a number line (either vertical or horizontal) or identify the values of given points on a number line. For example, students are able to identify where the following numbers would be on a number line: -4.5, 2, 3.2, -3 3/5, 0.2, -2, 11/2. Number lines can be used to show numbers and their opposites. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids. **MP.1, MP.2, MP.4** should be emphasized.
6.NS.7a	Students identify the absolute value of a number as the distance from zero but understand that although the value of -7 is less than -3, the absolute vale (distance) of -7 is greater than the absolute value (distance) of -3. Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line. For example, -4½ < -2 because -4½ is located to the left of -2 on the number line.

6.NS.7b	Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than 'less than' or 'greater than'. For example, the balance in Sue's checkbook was -12.55. The balance in Ron's checkbook was -10.45. Since -12.55 < -10.45, Sue owes more than Ron. The interpretation could also be "Ron owes less than Sue".
6.NS.7c	Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. For example, |-7| can be interpreted as the -7 distance is from 0 which would be 7. Likewise |7| can be interpreted as the distance 7 is from 0 which would also be 7. In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of -900 feet, write |-900| = 900 to describe the distance below sea level.
6.NS.7d	When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, -24 is less than -14 because -24 is located to the left of -14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of -24 is greater than -14. For negative numbers, as the absolute value increases, the value of the number decreases. **MP.1, MP.2, MP.7** should be emphasized.
6.NS.8	Students find the distance between points whose ordered pairs have the same x-coordinate (vertical) and same y-coordinate (horizontal) Coordinates could also be in two quadrants. Distance can be found using a number line or by calculating absolute value to determine distance. Students graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals. **MP.1, MP.2, MP.4, MP.5, MP.7** should be emphasized.

UNIT VOCABULARY

compatible numbers	dimensional analysis	unit ratio
Commutative Property	reciprocal	quadrants
absolute value	negataive integer	rational number
bar notation	opposites	repeating decimal
integer	positive integer	terminating decimals

BIG IDEAS

ENDURING UNDERSTANDINGS

ESSENTIALS QUESTIONS

Choose a few questions based on the needs of your students

The meanings of each operation on fractions are consistent with the meanings of the operations on whole numbers. For example: It is possible to divide fractions without multiplying by the inverse or reciprocal of the second fraction. • When dividing by a fraction, there are two ways of thinking about the operation – partition and measurement which will lead to two different thought processes for division. • When we divide one number by another, we may get a quotient that is bigger than the original number, smaller than the original number or equal to the original number.	How can estimating be helpful? - What does it mean to multiply and divide fractions? • Why does the process of invert and multiply work when dividing fractions? • When I divide one number by another number, do I always get a quotient smaller than my original number? • When I divide a fraction by a fraction what do the dividend, quotient and divisor represent? • What kind of models can I use to show solutions to word problems involving fractions? • Which strategies are helpful when dividing multi-digit numbers? • Which strategies are helpful when performing operations on multi-digit decimals? • When are negative numbers used and why are they important? • Why is it useful for me to know the absolute value of a number? • When is graphing on the coordinate plane helpful? • How do I use positive and negative numbers in everyday life? • Where do I place positive and negative rational numbers on the number line? • How do I use positive and negative numbers to represent quantities in real-world contexts? • What are opposites, and how are opposites shown on a number line?

CONNECTIONS

In **Critical Focus Area #2,** students use the meaning of fractions, the meanings of multiplication and division and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
The work in the unit will mark the final opportunity for students to demonstrate fluency with the four operations with whole numbers and decimals.

Standards for Mathematical Practice (SMP)
MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure (Deductive reasoning)
MP.8 Look for and express regularity in repeated reasoning (Inductive Reasoning)

Understand ratio concepts and use ratio reasoning to solve problems

	CONTENT	SKILLS
6.RP.3	Use ratio reasoning to solve real-world and mathematical problems	Use ratio reasoning to solve real-world and mathematical problems 1. Use a ratio to compare two different types of measure 2. Identify the conversion factor (e.g., 12 in. / 1 ft.) 3. Recognize that the conversion factor is a fraction equal to 1 (e.g., since the quantity described in numerator and denominator is the same) 4. Use ratios as conversion factors and the Identity Property of Multiplication to convert ratio units

Apply and extend previous understandings of multiplication and division to divide fractions by fractions		
	CONTENT	**SKILLS**
6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions. 1. Use visual models to divide whole numbers by fractions in word problems 2. Use visual models to divide fractions by fractions in word problems 3. Make drawings that represent understandings of division of fractions in word problems 4. Use equations and procedures to demonstrate understanding of division of fractions in word problems 5. Interpret and justify solutions to word problems 6. Write contextual problems for fraction division problems.
Compute fluently with multi-digit numbers and find common factors and multiples		
	CONTENT	**SKILLS**
6.NS.2	Fluently divide multi-digit numbers using the standard algorithm	Fluently divide multi-digit numbers using the standard algorithm 1. Use place value language when using the standard algorithm in division 2. Accurately use the standard algorithm for division of multi-digit numbers by multi-digit numbers 3. Efficiently use the standard algorithm for division of multi-digit numbers by multi-digit numbers
6.NS.3	Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.	Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation. 1. Use estimation strategies to support understanding of the addition and subtraction of decimals (e.g., first estimate the sum or difference, state whether the estimate will be higher or lower and then find the exact sum or difference) 2. Use place value language when using the standard algorithm in addition and subtraction of decimals. 3. Accurately use the standard algorithm in addition and subtraction of decimals 4. Efficiently use the standard algorithm in addition and subtraction of decimals 5. Use understanding of the patterns involved when multiplying and dividing by powers of ten to estimate products and quotients 6. Use place value language when using the standard algorithm in multiplication and division of decimals 7. Accurately use the standard algorithm in multiplication and division of decimals 8. Efficiently use the standard algorithm in multiplication and division of decimals
Apply and extend previous understandings of numbers to the system of rational numbers		
	CONTENT	**SKILLS**
6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values.	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. 1. Identify an integer and its opposite

6.NS.5	Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. 1. Use integers to represent quantities in real world situations 2. Explain where 0 fits into a situation represented by integers
6.NS.6	Understand a rational number as a point on a number line	Understand a rational number as a point on a number line 1. Identify a rational number as a point on the number line. 2. Identify the location of 0 on a number line in relation to positive and negative numbers 3. Recognize opposite signs of numbers as locations on opposite sides of 0 on the number line
6.NS.6	Extend number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates.	Extend number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates. 1. Recognize the signs of both numbers in an ordered pair indicate which quadrant of the coordinate plane the ordered pair will be located. 2. Find and position integers and other rational numbers on a horizontal or vertical number line diagram 3. Find and position integers and other rational numbers on a coordinate plane 4. Reason that the opposite of the opposite of a number is the number itself. 5. Reason that when only the x value in a set of ordered pairs are opposites, it creates a reflection over the y-axis 6. Recognize that when only the y value in a set of ordered pairs are opposites, it creates a reflection over the x-axis 7. Reason that when two ordered pairs differ only by signs, the locations of the points are related by reflections across both axes
6.NS.7	Understand ordering and absolute value of rational numbers	Understand ordering and absolute value of rational numbers 1. Order rational numbers on a number line 2. Identify absolute values of rational numbers 3. Interpret statements of inequality as statements about relative position of two numbers on a number line diagram 4. Write, interpret, and explain statements of order for rational numbers in real-world contexts 5. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation 6. Distinguish comparisons of absolute value from statements about order and apply to real world contexts

6.NS.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. 1. Calculate absolute value 2. Graph points in all four quadrants of the coordinate plane 3. Solve real-world problems by graphing points in all four quadrants of a coordinate plane 4. Given only coordinates, calculate the distances between two points with the same first coordinate or the same second coordinate using absolute value
UNIT RESOURCES		
Common Core Model Curriculum McGraw-Hill, **Glencoe Math**, Grade 6 Chapters 3-5 Georgia Math frameworks, Grade 6 Unit 1 & Unit 7 Manipulatives Smart Board resources Hands-On Standards		